

# A Mathematical Theory of Semantic Meaning

Mauro Guazzo\*

17th February 2011

---

\*Codework, Strada Pino Torinese 17-6, 10020 Baldissero Torinese (TO) Italy - ©Mauro Guazzo 2010 - Email: mauro.guazzo@gmail.com

**Contents**

<b>1</b>	<b>Delimiting the problem</b>	<b>2</b>
<b>2</b>	<b>Mutual Information</b>	<b>2</b>
<b>3</b>	<b>Model <math>S_0</math> - measuring consensus</b>	<b>3</b>
3.1	Summary of the Axioms in $S_0$ . . . . .	4
3.2	Limits of meaning in $S_0$ . . . . .	4
<b>4</b>	<b>Model <math>S_1</math> - information on the object</b>	<b>4</b>
<b>5</b>	<b>Model <math>S_2</math> - guessing the rules</b>	<b>5</b>
5.1	Spontaneous generation of meaning . . . . .	5
<b>6</b>	<b>Conclusions</b>	<b>5</b>

### Abstract

This work shows how to capitalize on Information Theory concepts to obtain an *experimental* measure of semantic meaning. This applies to artificial and natural languages alike. The key idea is to study a very simple classification model and regard it as a protolanguage. Many of the results that can be proven on a protolanguage can also be reasonably applied to the much vaster domain of a natural language.

## 1 Delimiting the problem

The problem of why human language carries a meaning, where meaning resides and how it can be measured is a formidable one indeed. This section discusses a few assumptions that we will use to make the very simplest cases amenable to a solution.

- Written language

For the sake of simplicity, assume we only deal with written language, that is, strings of characters. The passage from acoustic waveforms to characters is of course a huge simplification and impoverishment. But also a welcome one, if we agree that nothing essential is lost and the mystery of meaning is intact in written language.

- The concepts

We postulate that meaning resides in human brains and consists in some sort of correlation between the perception of an object and the words (or sentences or messages) that are associated with it. This mysterious link can be called a *concept*. A concept must work both ways: in the presence of an object, it acts like a decision rule that chooses a word — in the presence of a word it conjures the perception of the object.

Since a deterministic model of the brain of each speaker (in terms of firing neurons, axons, synapses, neurotransmitters, ...) is utterly out the question, this study leaves out brain and concepts and confines itself to keeping track of the text emitted by the speakers.

This choice has many consequences. The experimenter simply keeps a tally of emitted messages and is totally outside the phenomenon. He may not understand the language himself, so that he may hope to measure *how much* the messages mean, not *what* they mean.

Since the concepts are regarded as a *terra incognita*, *it is undecidable if a person speaks correctly in spite of having incorrect concepts*. This is important both as a foundation axiom of this approach and because it allows our model to apply smoothly to humans, animals, aliens, computers, irrespective of biological affinity.

- Language structure

Another huge difficulty is the great complexity of human languages. The (still undefined) meaning is probably associated with words, sentences and higher-level structures.

On the one hand we don't want to start off with a model entailing the whole structure of a natural language, on the other we don't wish to confine ourselves to totally primitive cases (like the classification of apples into 'small' 'medium' and 'large').

The proposed way out is to use units of text that we call messages. A message can be a single word, a phrase, a sentence or even garbled text. The point is that we take it as a whole and do not analyze its internal structure (at least in the initial phase of our study).

## 2 Mutual Information

We now recall a definition from Information Theory that we will need later.

Given a two-way table  $P$  of joint probabilities  $p_{ij}$  ( $i = 1, 2, 3, \dots, I$ ,  $j = 1, 2, 3, \dots, J$ ), define the marginal probabilities on rows and columns as

$$R_i = \sum_j p_{ij} \tag{1}$$

$$C_j = \sum_i p_{ij} \quad (2)$$

Then the mutual information of  $P$  is defined as

$$I_{XY}(P) = \sum_{i,j} p_{ij} \log_2 \frac{p_{ij}}{R_i \cdot C_j} \quad (3)$$

Terms where  $p_{ij} = 0$  are omitted from the sum.

Varying the  $p_{ij}$ , the mutual information varies from 0 to  $\log_2 \min(I, J)$

The quantity  $I_{XY}(P)$  is a measure of statistical dependence between rows and columns of  $P$ .

If you start off with vectors  $R_i, C_j$  and compute  $P$  as their *external product* then the resulting  $P$  will yield  $I_{XY}(P) = 0$ .

By external product of two vectors we mean to multiply the elements in all combinations. In this instance we obtain the two-way table as  $p_{ij} = R_i \cdot C_j$  for all  $i, j$ .

### 3 Model $S_0$ - measuring consensus

This section deals with the central problem of measuring how much meaning is contained in a very simple existing language.

We will call this model *Benchmark mode*, because it tries to measure meaning under objective experimental conditions.

An experimenter  $G$  intends to study the consensus of a population of speakers who are classifying objects.

A given stationary source  $O$  produces objects  $o_i$  (with  $i = 1, 2, 3, \dots, I$  and  $I \geq 2$ ).

A finite population  $Q$  of speakers  $q_j$  (with  $j = 1, 2, 3, \dots, J$  and  $J \geq 2$ ) is given.

A finite collection  $M$  of messages  $m_k$  (with  $k = 1, 2, 3, \dots, K$  and  $K \geq 2$ ) is also given.

Typically  $K$  is a small integer, whereas  $I$  and  $J$  stand for the size of large samples drawn from infinite populations.

We will call the set of these three components the *protolanguage* ( $QOM$ ).

The experiment consists in submitting each object  $o_i$  to each speaker  $q_j$ , who classifies it with one and only one of the messages  $m_k$  as the most appropriate.

Each speaker issues the message on the basis of his sensory perception (he looks at it, touches it, sniffs it, measures it,...)

The scenario is a general form of classification and we intend to measure the consensus among the speakers.

This work investigates the possibility of regarding this classification as a protolanguage, which can be either a simple artificial language (with made up messages) or else an aspect of a natural language.

Assuming now that we have an infinite number of speakers, the result of the experiment can be stored in a table  $A$  with  $I$  rows and  $K$  columns. Each row will contain the probabilities of the various messages for a given object.

The basic idea is to compare the message of each speaker with that of every other speaker and to compute a square table of co-occurrences.

To do this, for each row  $i$  of  $A$  we compute a square table  $a_i$  as the external product of the row with itself.

Then we compute the average

$$\bar{a} = \frac{1}{I} \sum_i a_i \quad (4)$$

The mutual information  $I_{XY}(\bar{a})$  is the proposed measure of meaning. As seen, it may range from zero to  $\log_2 K$ .

◇ Note that the mutual information of each  $a_i$ , that is,  $I_{XY}(a_i)$ , is zero for every  $i$  (because the table  $a_i$  is obtained via external product).

Similarly, if all rows of  $A$  are equal to the first row, then  $I_{XY}(\bar{a}) = 0$ . This is true even if the first row represents perfect consensus (contains all zeroes except a one).

In words:  $I_{XY}(\bar{a})$  measures consensus but also requires that consensus move to a different message when we move from an object to the next.

In still other words: the objects  $o_i$  are the reality that makes this kind of correlation (between messages) possible.

### 3.1 Summary of the Axioms in $S_0$

Axiom 0 **The classification is reproducible.**

The same object, shown to the same speaker at a different time, will produce the same message.

Axiom 1 **A speaker emits his message in isolation.**

Axiom 2 **The experimenter knows nothing about the speakers, except their messages.**

In particular, he does not know which aspects of the object the speaker considers or measures and which decision rules he follows. The concepts are beyond investigation.

Axiom 3 **The experimenter is outside the phenomenon.**

He may not be a competent speaker and may even be unable to examine or perceive or distinguish the objects.

Axiom 4 **The protolanguage is *symbolic*** (aka: conventional, arbitrary).

The messages could be expressed with different words, in a different alphabet or script and our setup would function equally well, yielding the same measure of meaning.

### 3.2 Limits of meaning in $S_0$

If the setup of model  $S_0$  is accepted, one must also accept that there can be no meaning, in the specified sense, if one of these conditions applies:

1. We don't have a plurality of speakers, of objects and of messages.

The second and third cases are straight-forward. The first is slightly different. We have treated the case of an infinite number of speakers. When the speakers are few, it is best to compare a speaker's choice with all others except himself, to avoid a bias. In the case of a single speaker the model fails for lack of data.

Our model of a protolanguage requires the three components ( $QOM$ ) and, in a sense, is the simplest possible model.

2. The speakers have no way to sensorially perceive the same objects.
3. The source  $O$  repeats itself indefinitely (or, more generally, the histogram of the message probabilities is the same for all objects).
4. There is zero consensus among the speakers.
5. The speakers had no chance to learn the protolanguage.

## 4 Model $S_1$ - information on the object

The setup of model  $S_0$  can be re-interpreted by interchanging an input and an output.

Suppose you carried out experiment  $S_0$  and succeeded in measuring a non-zero  $I_{XY}$ .

In the new setup  $S_1$ , a speaker is not allowed to perceive the object but he knows a message emitted by another speaker who did perceive it.

This setup can be regarded as the main use of a protolanguage, where the message provides information on the object — hence it will be called *Communication mode*.

To complete the experimental setup, suppose you ask a speaker to fetch an object that, in his view, corresponds to the message and then ask other speakers how they would classify the object.

The setup is nearly the same as under  $S_0$ , the difference being that in  $S_0$  you pick an object at random, whereas in  $S_1$  you pick a message at random (chosen with the appropriate probabilities), ask a speaker to fetch an object that he considers appropriate and then proceed as in  $S_0$ .

Hence we define that *the mutual information  $I_{XY}$  measured in  $S_0$  also measures the information given by the message on the object.*

What happens in a human mind on hearing a word (as, say, ‘wolf’) is still another matter, but we agreed to leave it out of our analysis.

## 5 Model $S_2$ - guessing the rules

There is another way to interchange an input and an output in model  $S_0$ .

In the third setup, which will be called *Learning-by-example mode*, a speaker both perceives the object and knows the relevant messages and uses this redundant information to update his linguistic competence (the concepts).

Obviously, in learning mode  $S_2$  we must relax Axiom 0: this will hold true when learning is complete.

What happens in the speaker’s mind ? Why do some speakers learn the language and others fail? All this is a mystery. We can only observe when the miracle did happen.

A very remarkable feature of Model  $S_2$  is that it does not require any priming (such as previous concepts, a common convention, a shared language).

### 5.1 Spontaneous generation of meaning

We now discuss the *Schuttrange experiment*, which was carried out several times with human speakers.

The experimenter designs an artificial protolanguage where the messages are (say, 3 to 6) fictitious words.

He then defines a collection of objects, for example the figures of an encyclopedia.

The volunteers (say, 5 to 10 of them) are instructed not to communicate except with the messages of the artificial language and to emit one and only one for each figure, while trying to gradually understand their correct usage.

In the initial phase of the experiment (say, the first 10 figures, that represent the training set) the experimenter shows a figure and announces the corresponding message.

In a second phase the experimenter keeps silent and the volunteers emit the messages (while hearing the other volunteers’ choice and interacting with the rest of the group).

The experimenter confines himself to record the choices and, at the end (after, say, 100 images), establishes that  $I_{XY} > 0$  and also reveals that the training set was in fact random.

The conclusion is that, if the human speakers are motivated, *the mere sustained effort of aiming at a consensus produces a meaning.*

## 6 Conclusions

This study intended to show that a very simplified model of semantic meaning allows for its experimental measurement.

Experimental proofs of semantic meaning are not new, but they were based on a different paradigm: the correct execution of a command-type sentence is taken to be a proof of its comprehension.

This model proposes a totally new paradigm, based on assertion-type sentences, mainly in the form of a simple classification.

Our model turns out to be very crude, if compared to the complexity of a human language, but many of its aspects and conclusions can reasonably be extrapolated to natural languages.

## References

- [1] M. Guazzo *Notes on a Model of Semantic Meaning* July 2010.

Note. The material of the present study is extracted from this book.

See [www.codework.it/semantic.htm](http://www.codework.it/semantic.htm) for its availability on internet.

The book takes a more general approach and also deals with the internal representation of meaning, artificial speakers, ways to model natural languages plus scattered notes on language and meaning.

- [2] C. E. Shannon *Collected Papers* Edited by N.J.A. Sloane, A.D. Wyner, IEEE Press 1993
- [3] R. G. Gallager *Information Theory and Reliable Communication* John Wiley 1968  
Mathematically oriented readers can find the basic ideas of Information Theory discussed in the first pages of this book or any similar textbook.
- [4] T. M. Cover, J. A. Thomas *Elements of Information Theory* Wiley 1991
- [5] R. Carnap *Introduction to Semantics* Cambridge, Harvard University Press 1942  
Previous approaches to semantic meaning can be found in countless treatises. Carnap and the next references are just starting points.
- [6] G. Bateson *Steps to an Ecology of Mind* San Francisco, Chandler 1972
- [7] N. Chomsky *Language and Mind* New York, Harcourt, Brace and World 1968
- [8] C. K. Ogden, I. A. Richards *The Meaning of Meaning. A Study of the Influence of Language upon Thought and of the Science of Symbolism* London, Routledge and Kegan 1923
- [9] F. R. Palmer *Semantics* Cambridge University Press 1981